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AN EOQ MODEL WITH RAMP TYPE DEMAND RATE, THREE PARAMETER WEIBULL DISTRIBUTION DETERIORATION AND SHORTAGES WITH PARTIAL BACKLOGGING

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ABSTRACT: In this paper, we have developed an order level inventory system for deteriorating items. Deterioration is assumed to follow three parameter Weibull distributions. The demand rate is linear function of time in beginning of cycle, which becomes constant as passage of time. Shortages are allowed and partially backlogged. Backlogging rate is variable which depends upon the duration of waiting time up to the arrival of next lot. Numerical example is used to illustrate the model. Sensitivity analysis is also performed to study the effect of change in various parameters on the behavior of the model.

KEYWORDS: Inventory, Deterioration, Partial backlogging, Ramp type demand rate

1. INTRODUCTION

Inventory is an important part of our manufacturing, distribution and retail infrastructure where demand plays an important role in choosing the best inventory policy. Researchers were engaged to develop the inventory models assuming the demand of the items to be constant, linearly increasing or decreasing, exponential increasing or decreasing with time. Inventory models with time-dependent demand were studied by Dave and Patel (1981) and Maiti et al. (2009).

Later, it has been realized that the above demand patterns do not precisely depict the demand of certain items such as newly launched fashion items, garments, cosmetics, automobiles etc, for which the demand increases with time as they are launched into the market and after some time, it becomes constant. In order to consider demand of such types, the concept of ramp-type demand is introduced. Ramp-type demand depicts a demand which increases up to a certain time after which it stabilizes and becomes constant. Mandal and Pal (1998) have developed inventory models with ramp type demand rate for deteriorating items. Panda et al. (2008) have developed optimal replenishment policy for perishable seasonal products taking ramp-type time dependent demand rate. Avanade et al. (2013) have developed considered demand function sensitive to price and time. Models for seasonal products with ramp-type time-dependent demand are discussed by Wang and Huang (2014).

Deterioration is defined as decay, damage, spoilage, evaporation, obsolescence, pilferage and loss of utility or loss of marginal value of a commodity that reduces usefulness from original ones. Blood, fish, fruits and vegetables, alcohol, gasoline, radioactive chemicals, medicines, etc., lose their utility with respect to time. In this case, a discount price policy is implemented by the suppliers of these products to promote sales. Thus, decay or deterioration of physical goods in stock is a very realistic feature. Modelers felt the need to take this factor into consideration. Various types of order-level inventory models for items deteriorating at a constant rate were discussed by Shah and Jaiswal (1977) and Dave (1986). As time progressed, several researchers developed inventory models with variable deterioration rate. In this connection, researchers may consult the work by Covert and Philip (1973), Chakrabarti et al. (1998), Jalan et al. (1996) and Dye (2004), who has used Weibull distribution for representing deterioration rate. Manna and Chaudhuri (2006) have developed an EOQ model with ramp type demand rate and time dependent deterioration rate. An inventory system with Markovian demands, phase type distributions for perishability and replenishment is developed by Chakravarthy (2011). San-José et al. (2014) have studied inventory system with partial backlogging and mixture of dispatching policies.

When shortage for a product occurs, some customers will go away, while some would like to wait for backlogging after the next replenishment. But the willingness is diminishing with the length of the waiting time for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be. Thus practically, all shortages are not backlogged but only some part of shortages is backlogged. This phenomenon is called partial backlogging.

Chang and Dye (1999) developed an inventory model in which the demand rate is time dependent and shortages are partially backlogged. Papachristos and Skouri (2000) developed an EOQ inventory model with time-dependent partial backlogging. Teng et al. (2003) then extended the backlogged demand to any decreasing function of the waiting time up to the next replenishment. The related analysis on inventory systems with partial backlogging have been performed by Teng and Yang (2004), Dye et al. (2006) etc. Singh and Singh (2007, 2009) studied inventory model with partial backlogging considering quadratic demand and power demand. San-Jose et al. (2015) have studied partial backlogging with non linear holding cost.

In this paper, an effort has been made to analyse an EOQ model for time-dependent deteriorating items assuming the demand rate to be a combination a linear and constant function of time. Such type of the demand pattern is generally seen in the case of any new brand of consumer goods coming to the market. The demand rate for such items increases with time up to certain time and then ultimately stabilizes and becomes constant. It is believed that such type of demand rate is quite realistic.

2. ASSUMPTIONS AND NOTATIONS:

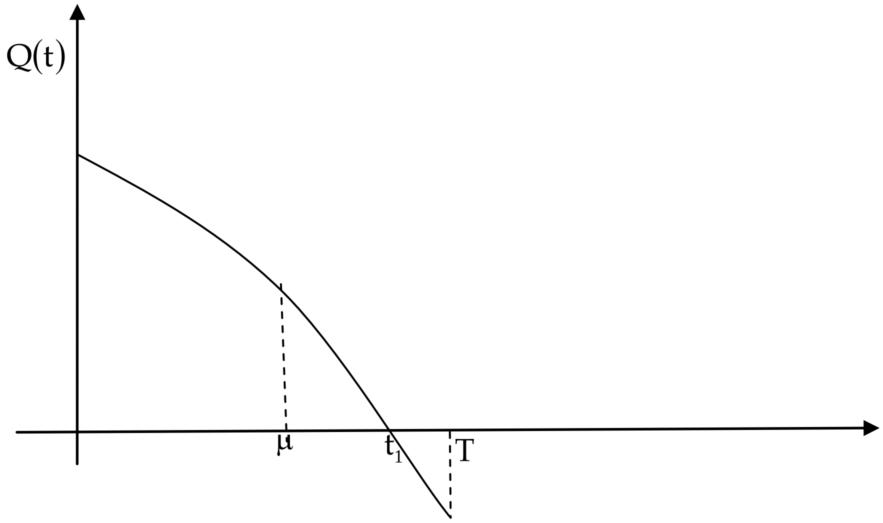
To develop an inventory model with variable demand and partial backlogging, the following notations and assumptions are used:

i) Replenishment is instantaneous and lead time is zero
ii) c_1 is the inventory holding cost per unit per unit of time.
iii) c_3 is the shortage cost per unit per unit of time.
iv) c_4 is the unit cost of lost sales.
v) c_5 is the cost of each unit.
vi) Ordering cost is c'.
vii) Demand rate is a combination a linear and constant function of time defined by
$f(t) = a\{t - (t-\mu)H(t-\mu)\}$ where a & μ are constants and $H(t-\mu)$ is Heviside's function defined as follows: $H(t-\mu) = \begin{cases} 0, t < \mu \\ 1, t \geq \mu \end{cases}$
Thus demand can be written as $f(t) = \begin{cases} at, t < \mu \\ a\mu, t \geq \mu \end{cases}$
viii) Unsatisfied demand is backlogged at a rate $u^{-\lambda t}$, where t is the time up to next replenishment and λ is a positive constant.
ix) R is the total cost per production cycle and T is the time for each cycle.
x) $Q(t)$ be the inventory level at time t.
xi) The distribution of the time to deterioration of the items follows three parameter Weibull distributions. Thus a variable fraction $\theta(t) = \alpha(t-\gamma)^{\beta-1}$, $(0 < \alpha \ll 1, t \geq 0)$ is the deterioration rate.
xii)

3. FORMULATION AND SOLUTION OF THE MODEL:

At the start of the cycle, the inventory level reaches its maximum S units of item at time t=0. During the time interval$[0, t_1]$, the inventory depletes due mainly to demand and partly to deterioration. At time $t = \mu < t_1$, the inventory level depletes and at t_1, the inventory level is zero and all the demand hereafter (i.e. $T - t_1$) is partially backlogged. The demand varies with time up to a certain time and become constant thereafter. The deterioration rate is described by an increasing function of time $\theta(t) = \alpha(t-\gamma)^{\beta-1}$.

A Graphical representation of the considered inventory system is given below:



The differential equations governing the instantaneous states of $Q(t)$ in the interval $[0, T]$ are as follows:

$$\frac{dQ(t)}{dt} + \theta(t)Q(t) = -f(t), 0 \leq t \leq \mu \quad (1)$$

$$\frac{dQ}{dt} + \theta(t)Q(t) = -f(t), \mu \leq t \leq t_1 \quad (2)$$

$$\frac{dQ}{dt} = -f(t)\, u^{-\lambda t} \;, t_1 \leq t \leq T \quad (3)$$

Conditions are $Q(0) = S$, $Q(t_1) = 0$

The solutions of equations (1) to (3) are given below:

$$Q(t) = e^{-\alpha(t-\gamma)^{\beta}}\left[e^{\alpha(-\gamma)^{\beta}}S + \frac{a\alpha(-\gamma)^{1+\beta}\gamma}{(1+\beta)(2+\beta)} - a\left\{\frac{t^2}{2} + \frac{\alpha(t-\gamma)^{1+\beta}(t+t\beta+\gamma)}{(1+\beta)(2+\beta)}\right\}\right],\; 0 \leq \; t \leq \mu \quad (4)$$

$$Q(t) = -\frac{ae^{-\alpha(t-\gamma)^{\beta}}\alpha\mu(\gamma-t_1)(-\gamma+t_1)^{\beta}}{1+\beta} - \frac{ae^{-\alpha(t-\gamma)^{\beta}}\mu\{t+t\beta+t\alpha(t-\gamma)^{\beta}-\alpha(t-\gamma)^{\beta}\gamma-t_1-\beta t_1\}}{1+\beta},\; \mu \leq t \leq t_1 \quad (5)$$

$$Q(t) = \frac{a(u^{-t\lambda}-u^{-\lambda t_1})\mu}{\lambda \text{Log} u},\; t_1 \leq t \leq T \quad (6)$$

Using above relations, S is given by

$$S = \frac{1}{2(1+\beta)(2+\beta)} ae^{-\alpha(-\gamma)^{\beta}}\left[(1+\beta)(2+\beta)\mu(-\mu+2t_1) + 2\alpha\left\{-(-\gamma+\mu)^{2+\beta} + (2+\beta)\mu t_1(-\gamma+\right.\right.$$
$$t1\beta + -\gamma\beta\gamma 2 - 2 + \beta\mu\gamma(-\gamma + t1)\beta \quad (7)$$

The inventory holding cost during the interval $(0, T)$ is given by

$$C_H = c_1\left[\int_0^{\mu} Q(t)dt + \int_{\mu}^{t_1} Q(t)\,dt\right]$$

$=\frac{c_1}{6(1+\beta)(2+\beta)(3+\beta)}\Big[6e^{\alpha(-\gamma)^\beta}S(2+\beta)(3+\beta)\{\alpha(-\gamma)^{1+\beta}+\mu+\beta\mu-\alpha(-\gamma+\mu)^{1+\beta}\}+$

$\alpha-(1+\beta)(2+\beta)(3+\beta)\mu 3+6\alpha-\gamma 2+\beta 3\gamma-3+\beta\mu+3\alpha-\gamma+\mu 1+\beta 6\gamma 2+4\beta\gamma\mu+\beta(1+\beta)\mu 2$

$+c_1 a\mu\Big[\frac{\mu^2}{2}-\frac{2\gamma(-\gamma+\mu)^{1+\beta}}{2+3\beta+\beta^2}-\frac{\beta\mu(-\gamma+\mu)^{1+\beta}}{2+3\beta+\beta^2}-\mu t_1+\frac{2(-\gamma+\mu)^{1+\beta}t_1}{2+3\beta+\beta^2}+\frac{\beta(-\gamma+\mu)^{1+\beta}t_1}{2+3\beta+\beta^2}+\frac{t_1^2}{2}-\frac{\mu(-\gamma+t_1)^{1+\beta}}{1+\beta}+$

$t1(-\gamma+t1)\,1+\beta 1+\beta-2(-\gamma+t1)\,2+\beta 2+3\beta+\beta 2$ (8)

The cost due to deterioration of units in the period $(0,T)$ is given by

$C_D = c_5$ (Initial inventory level-Total units sold)

$$= c_5\left[S-\int_0^{t_1} f(t)\,dt\right]$$

$= c_5\Big[-\frac{a\mu^2}{2}-a\mu(-\mu+t_1)+\frac{1}{2(1+\beta)(2+\beta)}ae^{-\alpha(-\gamma)^\beta}\{(1+\beta)(2+\beta)\mu(-\mu+2t_1)+$

$2\alpha--\gamma+\mu 2+\beta+2+\beta\mu t1-\gamma+t1\beta+-\gamma\beta\gamma 2-2+\beta\mu\gamma-\gamma+t1\beta$ (9)

The cost due to shortages in the interval $(0,T)$ is given by

$$C_S = -c_3\left[\int_{t_1}^{T} Q(t)\,dt\right]$$

$$= -c_3\left(-\frac{au^{-T\lambda}\mu}{\lambda^2\text{Log}[u]^2}+\frac{au^{-\lambda t_1}\mu}{\lambda^2\text{Log}[u]^2}-\frac{aTu^{-\lambda t_1}\mu}{\lambda\text{Log}[u]}+\frac{au^{-\lambda t_1}\mu t_1}{\lambda\text{Log}[u]}\right) \quad (10)$$

The opportunity cost due to lost sales in the interval $(0,T)$ is given by

$C_O = c_4\left[\int_{t_1}^{T}(1-u^{-\lambda t})f(t)dt\right]$

$= a\mu c_4\left(T+\frac{u^{-T\lambda}-u^{-\lambda t_1}}{\lambda\text{Log}[u]}-t_1\right)$ (11)

The total cost R in the system in the interval $(0,T)$ is given by

$R = c' + C_H + C_D + C_S + C_O$ (12)

In above relation, c' is constant, while $C_H, C_D, C_S \& C_O$ are given by the equations (8) to (11).

The average cost K in the system in the interval $(0,T)$ is given by

$K=\frac{R}{T}$ (13)

The optimum values of t_1 and T which minimize average cost K are obtained by using the equations:

$$\frac{\partial K}{\partial t_1}=0 \text{ and } \frac{\partial K}{\partial T}=0\,,$$

Now,

$\frac{\partial K}{\partial t_1}=0$

$\Rightarrow$

$$a(-1+u^{-\lambda t_1})\mu c_4 - a\mu c_5 - au^{-\lambda t_1}\mu c_3(T-t_1) + \frac{a\mu c_1[-\mu-\beta\mu-\gamma(-\gamma+\mu)^\beta+\mu(-\gamma+\mu)^\beta+t_1+\beta t_1+(-\gamma+t_1)^\beta\{\gamma-(1+\beta)\mu+\beta t_1\}]}{1+\beta}=0 \quad (14)$$

Also, $\frac{\partial K}{\partial T}=0$ gives

$$c' + \frac{aT(u^{-T\lambda}-u^{-\lambda t_1})\mu c_3}{\lambda \mathrm{Log}[u]} - \mathrm{Ta}(1-u^{-T\lambda})\mu c_4 + a\mu c_4\left(T + \frac{u^{-T\lambda}-u^{-\lambda t_1}}{\lambda \mathrm{Log}[u]} - t_1\right) - c_3\left(-\frac{au^{-T\lambda}\mu}{\lambda^2 \mathrm{Log}[u]^2} +\right.$$

$au-\lambda t1\mu\lambda 2\mathrm{Log}[u]2-aTu-\lambda t1\mu\lambda\mathrm{Log}[u]+au-\lambda t1\mu t1\lambda\mathrm{Log}[u]$

$+c_1\left[\frac{1}{6(1+\beta)(2+\beta)(3+\beta)}\left\{6e^{\alpha(-\gamma)^\beta}S(2+\beta)(3+\beta)\left(\alpha(-\gamma)^{1+\beta}+\mu+\beta\mu-\alpha(-\gamma+\mu)^{1+\beta}\right)+a\left((-1-\right.\right.\right.$

$\beta 2+\beta 3+\beta\mu 3+6\alpha-\gamma 2+\beta 3\gamma-3+\beta\mu+3\alpha-\gamma+\mu 1+\beta 6\gamma 2+4\beta\gamma\mu+\beta 1+\beta\mu 2+a\mu\mu 22-2\gamma(-\gamma+\mu)1+\beta 2+3\beta$
$+\beta 2-\beta\mu(-\gamma+\mu)1+\beta 2+3\beta+\beta 2-\mu t1+2(-\gamma+\mu)1+\beta t12+3\beta+\beta 2+\beta(-\gamma+\mu)1+\beta t12+3\beta+\beta 2+t122-\mu$
$(-\gamma+t1)1+\beta 1+\beta+t1(-\gamma+t1)1+\beta 1+\beta-2(-\gamma+t1)2+\beta 2+3\beta+\beta 2+c5-a\mu 22-a\mu-\mu+t1+121+\beta 2+\beta$
$ae-\alpha-\gamma\beta(1+\beta)(2+\beta)\mu(-\mu+2t1)+2\alpha-(-\gamma+\mu)2+\beta+(2+\beta)\mu t1(-\gamma+t1)\beta+\gamma((-\gamma)\beta\gamma-(2+\beta)\mu(-\gamma+$
$t1)\beta)=0$ (15)

4. NUMERICAL EXAMPLE

To illustrate the model numerically, we use the following parameter values:

$c_1 = 2.4,\ c_3 = 5,\ c_4 = 10,\ c_5 = 8,\ c' = 100, \mu = \frac{2}{3},\ \alpha = 0.002,\ \beta = 20,\ \gamma = 0.6,\ a = 9000,$
$u = e, \lambda = 0.1$

Applying the subroutine Find Root in Mathematical 8, we obtain the optimal solution for t_1 and T as follows:

$$t_1 = 1.66245, T = 1.77520$$

Also, the optimal average cost for these parameters is 12607.8

5. SENSITIVITY ANALYSIS

Sensitivity analysis is performed by changing (increasing and decreasing) the parameters by 10%, 30% and 50%, and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table1

Changing Parameter	% Change	t_1	T	S	Average Cost
c_1	+50	1.63924	1.91369	7836.72	17344.0
	+30	1.64756	1.85931	7886.88	15511.2
	+10	1.65708	1.80356	7944.31	13597.6
	−10	1.66834	1.74656	8012.33	11593.7
	−30	1.68225	1.68869	8096.49	9486.2
	−50	1.70072	1.63100	8208.60	7259.0

c'	+50	1.66248	1.77619	7976.90	12636.8
	+30	1.66247	1.77579	7976.84	12625.2
	+10	1.66246	1.77540	7976.77	12613.6
	−10	1.66245	1.77501	7976.71	12613.6
	−30	1.66243	1.77461	7976.65	12590.4
	−50	1.66242	1.77422	7976.58	12578.7
c_3	+50	1.66267	1.74303	7978.04	12647.5
	+30	1.66260	1.75339	7977.63	12634.6
	+10	1.66251	1.76685	7977.08	12618.0
	−10	1.66238	1.78503	7976.34	12595.8
	−30	1.66221	1.81096	7975.29	12564.5
	−50	1.66194	1.85097	7973.67	12517.2
c_4	+50	1.66346	1.63191	7982.83	12748.6
	+30	1.66301	1.68763	7980.14	12737.2
	+10	1.66262	1.74544	7977.79	12665.9
	−10	1.66229	1.80557	7975.80	12534.7
	−30	1.66203	1.86828	7974.22	12343.6
	−50	1.66185	1.93383	7973.11	12092.0
c_5	+50	1.68425	1.82174	8108.63	13454.2
	+30	1.67655	1.80422	8062.02	13123.0
	+10	1.66757	1.78532	8007.71	12782.6
	−10	1.65678	1.76450	7942.51	12429.1
	−30	1.64322	1.74074	7860.72	12055.9
	−5	1	1	7	1

	0	.62490	.71189	750.37	1648.8
α	+50	1.66246	1.77540	7977.79	12613.5
	+30	1.66245	1.77532	7977.37	12611.2
	+10	1.66245	1.77524	7976.95	12608.9
	−10	1.66245	1.77517	7976.53	12606.6
	−30	1.66245	1.77509	7976.11	12604.4
	−50	1.66245	1.77501	7975.69	12602.1
β	+50	1.64123	1.73881	7848.72	12048.4
	+30	1.64772	1.75002	7887.87	12222.3
	+10	1.65663	1.76529	7941.60	12456.9
	−10	1.66961	1.78730	8019.92	12790.6
	−30	1.69028	1.82180	8144.58	13303.1
	−50	1.72852	1.88363	8373.52	14192.3
γ	+50	1.94587	2.07640	9674.34	14452.5
	+30	1.83208	1.95519	8994.04	13711.5
	+10	1.71883	1.83487	8314.89	12972.8
	−10	1.60626	1.71589	7639.73	12246.3
	−30	1.49453	1.59846	6969.75	11534.9
	−50	1.38390	1.48286	6306.48	10841.3
μ	+50	1.68828	1.78114	10699.50	18197.3
	+30	1.67699	1.77893	9704.06	16056.1
	+10	1.66704	1.77655	8584.93	13785.0
	−10	1.65807	1.77371	7335.25	11406.7
	−30	1.64982	1.77025	5950.37	8945.5

	−50	1.64214	1.76625	4427.11	6427.1
u	+50	1.66314	1.67105	7980.91	12746.8
	+30	1.66288	1.70762	7979.32	12720.0
	+10	1.66260	1.75057	7977.63	12658.0
	−10	1.66230	1.80256	7975.84	12539.5
	−30	1.66201	1.86838	7974.06	12321.8
	−50	1.66177	1.95801	7972.61	11905.9
λ	+50	1.66333	1.64699	7982.03	12751.4
	+30	1.66295	1.69799	7979.73	12729.4
	+10	1.66260	1.74936	7977.67	12660.2
	−10	1.66231	1.80116	7975.89	12543.3
	−30	1.66207	1.85346	7974.42	12377.7
	−50	1.66188	1.90633	7973.30	12162.4

From Table 1, the following points are noted:

1. It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters except c_4 , β and μ
2. It is observed that the model is more sensitive for a negative change than an equal positive change in the parameter c_4, μ & β.
3. The optimal cost increases (decreases) and decreases (increases) with the increase (decrease) and decrease (increase) in the value of the parameters $c_1, c', c_3, c_4, c_5, \alpha, \gamma, \mu, u$ & λ but this trend is reversed for the parameter β.
4. Model is highly sensitive to changes in c_1, μ & γ and moderately sensitive to changes in c_5 & β . It has low sensitivity to c', c_3, c_4, α, u & λ.
5. From the above points, it is clear that much care is to be taken to estimate c_1, μ & γ.

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